Self-Excited Wave Oscillations in a Water Table

Wladimiro Calarese* and Wilbur L. Hankey†
Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio

An experimental investigation has been performed on self-excited wave oscillations on cavity, spike-tipped, and inlet models in a water table. Buzzing was generated by positioning the models at a small angle of attack with respect to the freestream flow. The hydraulic analogy was used to compare the results obtained in water to results obtained in a gas. High-speed and real-time photography were used in the experiment. The frequencies of oscillations in water and air were consistent with the hydraulic analogy. Numerical solutions of the phenomenon were also obtained.

Nomenclature

a	= speed of sound in air, or celerity in water
c	= wave speed (celerity)
C_p	= specific heat capacity at constant pressure
Fr	= Froude number
f	= oscillation frequency
g	= acceleration of gravity
h	= water depth
H	= enthalpy
L/D	= cavity length-to-depth ratio
M	= Mach number
p	= pressure
R	= gas constant
T	= temperature
u	= x velocity component
v	= y velocity component
U,V	= velocity
x,y	= Cartesian coordinates
α	= $2\pi\delta/\lambda$ dimensionless wave number
γ	= specific heat capacity ratio
δ	= shear-layer thickness
λ	= wavelength
ρ	= density
Φ	= disturbance amplitude

Subscripts

∞	= infinity
a	= air
i	= imaginary
o	= stagnation
r	= real
w	= water
x,y	= derivatives

Introduction

THE analogy between the flow of water with a free surface and the flow of a two-dimensional compressible gas (hydraulic analogy) has been shown in the past to be valid.[1-4] It is therefore advantageous at times to recreate in water (incompressible fluid) a physical phenomenon that occurs in a compressible gas and to make the appropriate comparisons. The advantage derives from better visualization capability and from considerable savings in energy and funds. It is in fact far more inexpensive and energy saving to operate a water table than a supersonic wind tunnel. In addition, some phenomena in water can be observed directly without sophisticated equipment, while in a wind tunnel it is necessary to make use of schlieren high-speed photography, holography, interferometry, etc. An experimental investigation was therefore performed on self-excited wave oscillations occurring on two-dimensional bodies in water. The objective was to study the "buzzing" phenomenon in water and to compare the results, whenever possible, with previous experimental results obtained in air. Necessary conditions for buzzing to occur are the presence of a shear layer with an inflection point in the velocity profile (separation), a reflecting surface (cavity wall, body shoulder, inlet cowl, etc.), and the appropriate body length to permit "in-phase" reflection of pressure waves. If these conditions are met, resonance occurs in the frequency range of the oscillations and amplification of the disturbance takes place until a "limit cycle" is reached. Photographic data were obtained, i.e., high-speed and real-time movies, to visually illustrate the phenomenon. Furthermore, a stability analysis was developed and numerical solutions were obtained to determine the conditions for instabilities and disturbances like buzzing to occur in water. Cavity, spike-tipped, and inlet models were used in the water table.

Equipment and Test Description

The water table of the Air Force Institute of Technology was used for the experiment. The table is 1.22×2.44 m and the water flow is regulated by a water pump with a pumping capacity of 100 liter/min. The tests were performed on different models: a rectangular cavity model, a blunt model with a protruding spike, and an external compression inlet model. The models were made of wood and waterproofed with Imron, a polyurethane enamel. The model dimensions and the models are shown in Figs. 1-4. The Froude number used, which is approximately equal to the Mach number, was 3 because of the availability of "buzzing" data obtained in airflow on inlets and spikes at Mach numbers between 2 and 3. In order to produce a Froude number of 3, converging-diverging nozzle blocks appropriately designed were positioned in the water table as shown in Figs. 2-4. The throat diameter was $D_t = 25.4$ cm for a water flow of 100 liter/min.

For most of the tests, the depth of the stagnating water upstream of the nozzle blocks was about 3.5 cm, while the water depth at the nozzle exit was 0.635 cm. Therefore, the freestream water velocity was 0.75 m/s, while the wave propagation velocity was 0.25 m/s. The models were positioned downstream of the nozzle blocks at various distances and inclinations with respect to the water flow. In order to obtain strong "buzzing," the cavity was positioned

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*Aerospace Engineer, Flight Dynamics Laboratory. Member AIAA.

†Senior Scientist, Flight Dynamics Laboratory. Member AIAA.

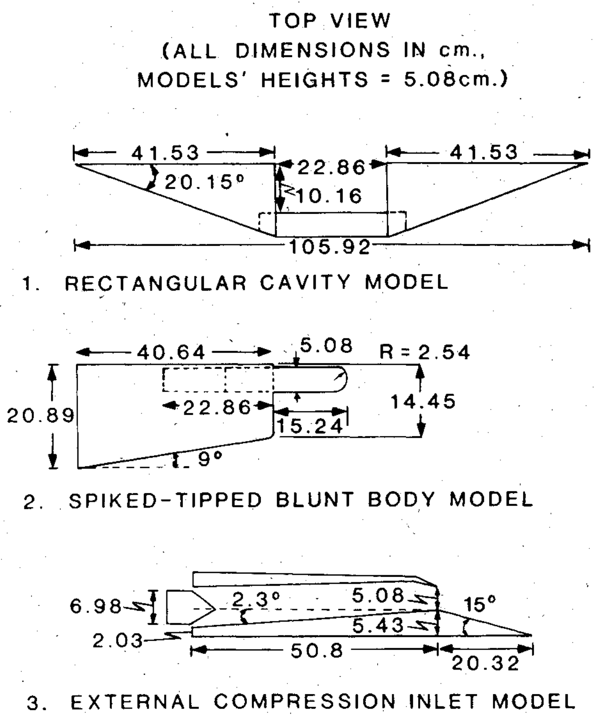

Fig. 1 **Model dimensions.**

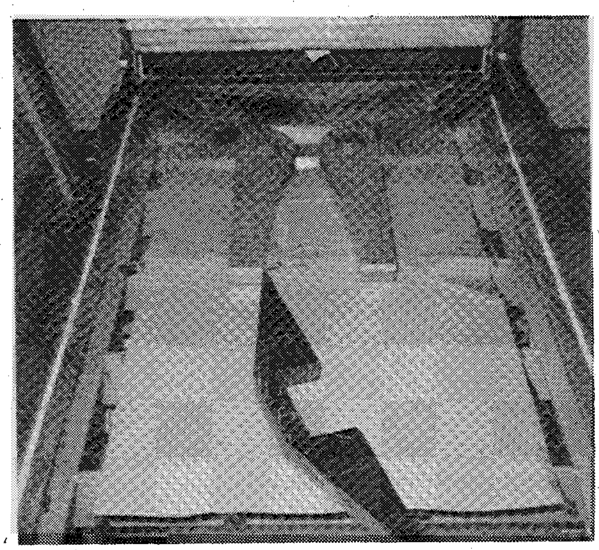
Fig. 2 **Rectangular cavity models.**

in contact with the nozzle at a compression angle of 16 deg, the inlet was positioned at the nozzle exit at an expansion angle of 12 deg, and the spike was positioned downstream of the nozzle blocks at an expansion angle of 12 deg. Buzzing for the cavity was also obtained with the model positioned at 0 deg angle with the water flow. Once the self-excited oscillations were established, three variable speed cameras were used, one each from the model's side, front, and rear. The dye used to enhance visualization was a water solution of potassium permanganate.

Theoretical Background

It is useful to illustrate the main features of the hydraulic analogy. The free surface water has a flow over a horizontal channel (if the channel is inclined, then the water velocity is proportional to the sine of the inclination angle), bounded by the walls which are similar to the walls of the corresponding compressible gas flow. The analogy can be obtained by using the conservation equations of mass, momentum, and energy

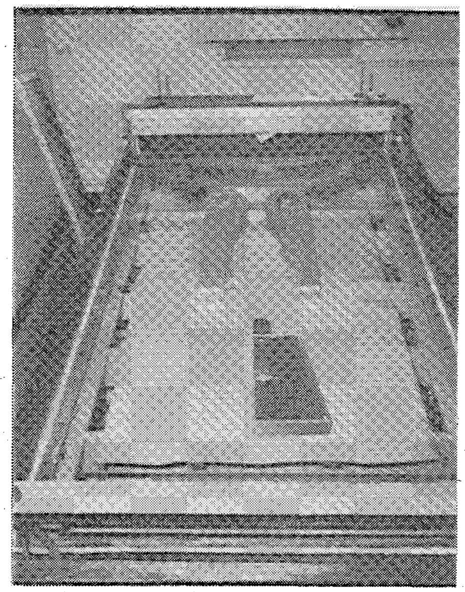
Fig. 3 **Spike-tipped blunt body model.**

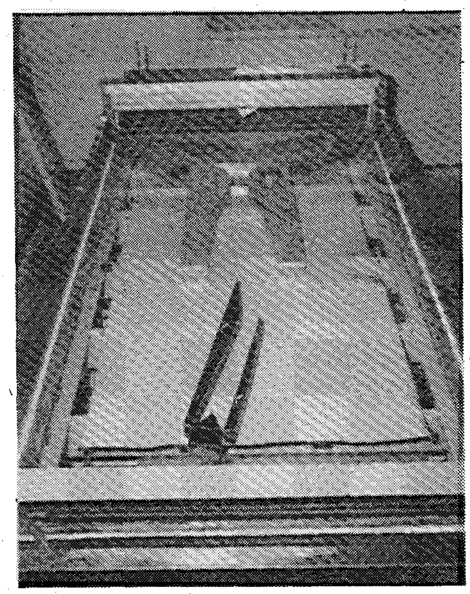
Fig. 4 **External compression inlet model.**

for both flows. From the energy equation for steady flow,

Gas	Water	
$C_pT + \frac{1}{2}V^2 = C_pT_o$	$gh + \frac{1}{2}V^2 = gh_o$	(1)
$V^2 = 2C_p(T_o - T)$	$V^2 = 2g(h_o - h)$	(2)
$V_{\max} = \sqrt{2C_pT_o}$	$V_{\max} = \sqrt{2gh_o}$	(3)

Therefore, equating $V/V_{\max}$ for gas and water,

$$\frac{V^2}{V_{\max}^2} = \frac{T_o - T}{T_o} = \frac{h_o - h}{h_o} \tag{4}$$

or

$$T/T_o = h/h_o \tag{5}$$

From the continuity equation,

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad \text{(gas)}$$
$$\frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \quad \text{(water)} \tag{6}$$

we get

$$\rho/\rho_o = h/h_o \tag{7}$$

and therefore, from Eq. (5)

$$\rho/\rho_o = h/h_o = T/T_o \tag{8}$$

From the isentropic relation for a gas

$$p/p_o = (\rho/\rho_o)^\gamma \tag{9}$$

If $\gamma = 2$, then

$$p/p_o = (\rho/\rho_o)^2 \tag{10}$$

and from Eq. (7)

$$p/p_o = (h/h_o)^2 \tag{11}$$

The speed of sound in a gas is

$$a = \sqrt{\gamma RT} = \sqrt{\gamma p/\rho} \tag{12}$$

From momentum considerations it can be shown that p in air is equivalent to $\frac{1}{2}gh^2$ in water and ρ to h. Using these values in Eq. (12)

$$a = \sqrt{\gamma gh/2} = \sqrt{gh} \tag{13}$$

since for the water analogy, $\gamma = 2$.

The Mach number in a gas is defined as

$$M = V/\sqrt{\gamma RT} \tag{14}$$

while the corresponding Froude number for water is defined as

$$Fr = V/\sqrt{gh} \tag{15}$$

If again $\gamma = 2$ for the gas, then the Mach number is analogous to the Froude number. Therefore, using $\gamma = 2$, all compressible flow equations derived for a gas are valid for water. For steady flow, the only condition for the analogy to hold is that the gas must possess $\gamma = 2$. The corresponding quantities of the hydraulic analogy are shown in Table 1.

It is easy to show that, for real gases, $Fr = \sqrt{\gamma/2}\, M$. In past experimental work, however, Fr has been set equal to M for airflow as a modification of the direct analogy. The subsonic/supersonic flow and shock waves in a gas correspond to streaming water, shooting water, and hydraulic jump, respectively. For gases, $\gamma \neq 2$, only the correspondence of the density ratio to water depth ratio holds exactly. If a test were to be performed in water, it would be feasible to use the depth ratio to get the density ratio and use the latter to get pressure and temperature ratios by means of the aerodynamic relations. Direct use of the depth ratio is not suggested. This is a limitation of the analogy. Other limitations exist such as the presence of capillary waves which complicate the evaluation of photographic material and impair the accuracy of the water depth measurements. Changes in velocities due to flow angles in water are greater than those obtained by the method of characteristics. Reynolds numbers in water are very low compared to air, so it is more important to have fairly close flow patterns than fairly close Reynolds numbers.

The hydraulic jump occurs only in shooting water since the water velocity must be greater than the wave propagation speed. The vertical deceleration through the jump is so great that it is no longer negligible with respect to gravity and, as a result, some distortions in the water flow pattern occur, e.g., the water depth would be lower than the depth expected from the hydrodynamic equations. The water depth increases through the hydraulic jump since, for an incompressible fluid, an increase in entropy corresponds to an increase of the internal energy of the fluid and a decrease in the energy of the flow. Therefore, an incompressible fluid (water) has a loss of energy (heat) through a jump while a compressible fluid (air) experiences no heat losses through a shock.

When oscillations are present as in the case of "buzzing," a stability analysis can be developed for the unsteady flow.

Table 1 Quantities of hydraulic analogy

Gas ($\gamma = 2$)		Water
Temperature ratio, T/T_o	→	h/h_o, depth ratio
Pressure ratio, p/p_o	→	$(h/h_o)^2$, depth ratio squared
Density ratio, ρ/ρ_o	→	h/h_o, depth ratio
Speed of sound, $a = \sqrt{2RT}$	→	$\sqrt{gh}$, celerity (wave speed)
Mach number, V/a	→	$V/\sqrt{gh}$, Froude number

Stability Analysis

In this section the stability of water flow with a free surface (encountered on a water table) will be investigated. The governing equations for incompressible, inviscid (but rotational) flow on a water table are as follows[4]:

Continuity

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 \tag{16}$$

Momentum

$$\frac{Du}{Dt} = -g\frac{\partial h}{\partial x} \tag{17}$$

$$\frac{Dv}{Dt} = -g\frac{\partial h}{\partial y} \tag{18}$$

The stability of a perturbed parallel flow with an arbitrary velocity profile will be examined. Let

$$u = \bar{u}(y) + u'(x,y,t) \tag{19}$$

$$v = 0 + v'(x,y,t) \tag{20}$$

$$h = h_\infty + h'(x,y,t) \tag{21}$$

Inserting these relationships into the governing equations and assuming small disturbance produces a linear set of equations for the first-order terms,

$$h'_t + \bar{u}h'_x + h_\infty(u'_x + v'_y) = 0 \tag{22}$$

$$u'_t + \bar{u}u'_x + \bar{u}_y v' + gh'_x = 0 \tag{23}$$

$$v'_t + \bar{u}v'_x + gh'_y = 0 \tag{24}$$

The solution of this set of equations is of the following form:

$$h' = \hat{h}(y)e^{i\alpha(x-ct)} \tag{25}$$

$$u' = \hat{u}(y)e^{i\alpha(x-ct)} \tag{26}$$

$$v' = \Phi(y)e^{i\alpha(x-ct)} \tag{27}$$

where

$$c = c_r + ic_i \tag{28}$$

Inserting these relationships into the small-disturbance equations and rearranging produces these three equations,

$$g\hat{h}_y = -i\alpha(\bar{u}-c)\Phi \tag{29}$$

$$\left[1-\left(\frac{\bar{u}-c}{a_\infty}\right)^2\right]i\alpha\hat{u} = \frac{(\bar{u}-c)\bar{u}_y}{a_\infty^2}\Phi - \Phi_y \tag{30}$$

$$\left[\frac{(\bar{u}-c)\Phi_y - \bar{u}_y\Phi}{1-\left(\frac{\bar{u}-c}{a_\infty}\right)^2}\right]_y = \alpha^2(\bar{u}-c)\Phi \tag{31}$$

where

$$a_\infty^2 = gh_\infty \tag{32}$$

Equation (31) is identical to the Rayleigh equation for investigating the stability of a shear layer in a compressible flow.[5] Further analysis indicates that the hydraulic analogy for unsteady flow is identical to a gas for the following situation:

Water	Gas ($\gamma = 2$)
$Fr = U_\infty / a_\infty$	$M = U_\infty / a_\infty$
$a_\infty^2 = gh_\infty$	$a_\infty^2 = \gamma R T_\infty$
$\bar{H}(y) = \frac{\bar{u}^2}{2} + gh_\infty$	$\bar{H}(y) = \frac{\bar{u}^2}{2} + \frac{\gamma}{\gamma-1}RT_\infty$

For the analogy to hold for unsteady flow, two conditions must be met, i.e., the gas must initially be isothermal with $\gamma = 2$.

Since Ref. 5 developed the relationships to numerically evaluate the eigenvalues in the Rayleigh equation for $\Phi(\alpha,c)$, this procedure can therefore be adopted here.

Numerical Solution

The stability of a parallel stream with an arbitrary velocity profile on a water table was shown to be identical to an isothermal gas with $\gamma = 2$. This flow can be described by Rayleigh's equation and the condition under which instabilities occur can be ascertained by numerically evaluating the eigenvalues.

A disturbance of v' is as follows

$$v' = \Phi(y)e^{i\alpha(x-ct)} \tag{33}$$

or

$$v' = \Phi e^{\alpha c_i t}[\cos\alpha(x-c_r t) + i\sin\alpha(x-c_r t)] \tag{34}$$

The v' disturbance will grow or decay depending upon the sign of c_i. Rayleigh[6] showed positive c_i results when the $\bar{u}(y)$ profile possesses an inflection point. A velocity profile of the following general shape was found to be useful for investigating these flows,

$$\bar{u}/u_\infty = \tfrac{1}{2}[1+\tanh(y/\delta)] \tag{35}$$

Insertion of $\bar{u}$ into the Rayleigh equation for Φ permits the evaluation of the eigenvalues for c at different Froude numbers. Boundary conditions are,

$$v'(y=-\infty) = 0 \tag{36}$$

$$h'(y=+\infty) = 0 \tag{37}$$

This implies

$$\Phi(\pm\infty) = 0 \tag{38}$$

Results of numerical calculations of the eigenvalues for the hyperbolic-tangent velocity profile are shown in Figs. 5 and 6. The Rayleigh instability is found to diminish with Froude number and nearly disappears at $Fr = 3.5$. For subcritical wave propagation velocity, only one large amplification mode of the disturbance is observed, while at supercritical propagation velocity, multiple modes are observed but the amplification factor is small and almost negligible.

Experimental Results

Cavity

The self-excited oscillations, seen as traveling hydraulic jumps, were generated by placing the cavity model with an

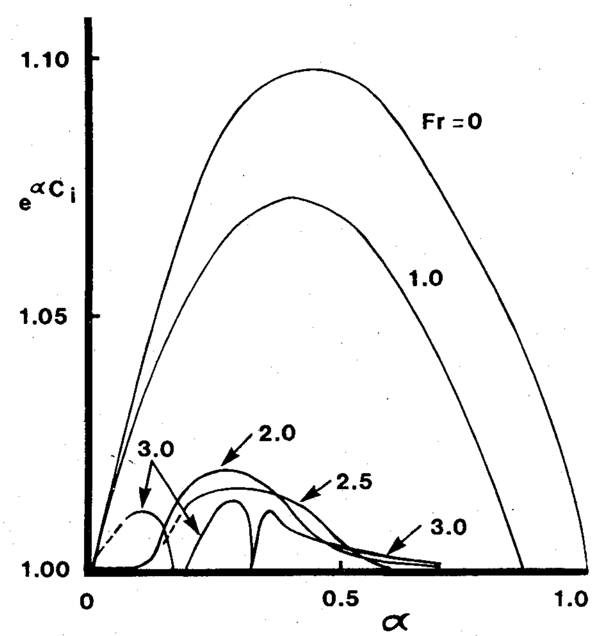

Fig. 5 Amplification factor vs wave number.

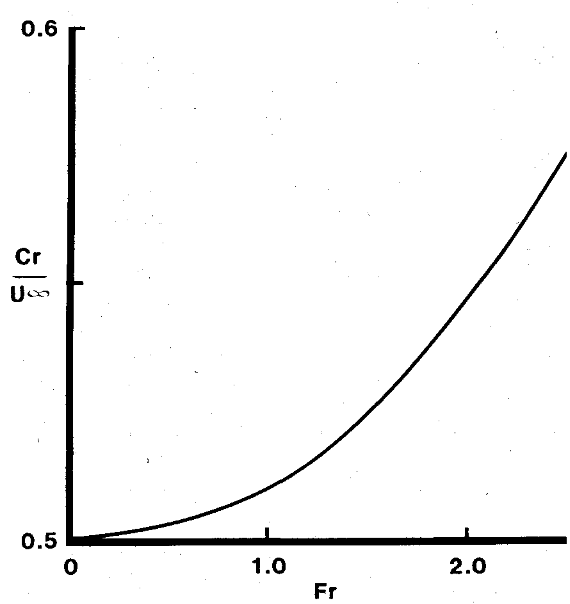

Fig. 6 Propagation velocity for traveling waves in a free shear layer.

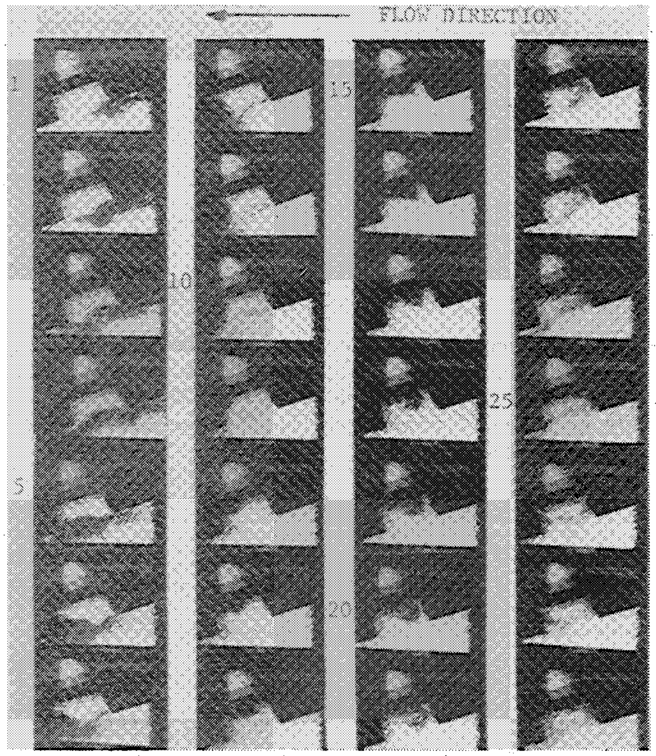

Fig. 7 Wave oscillations in a rectangular cavity. Dye upstream motion.

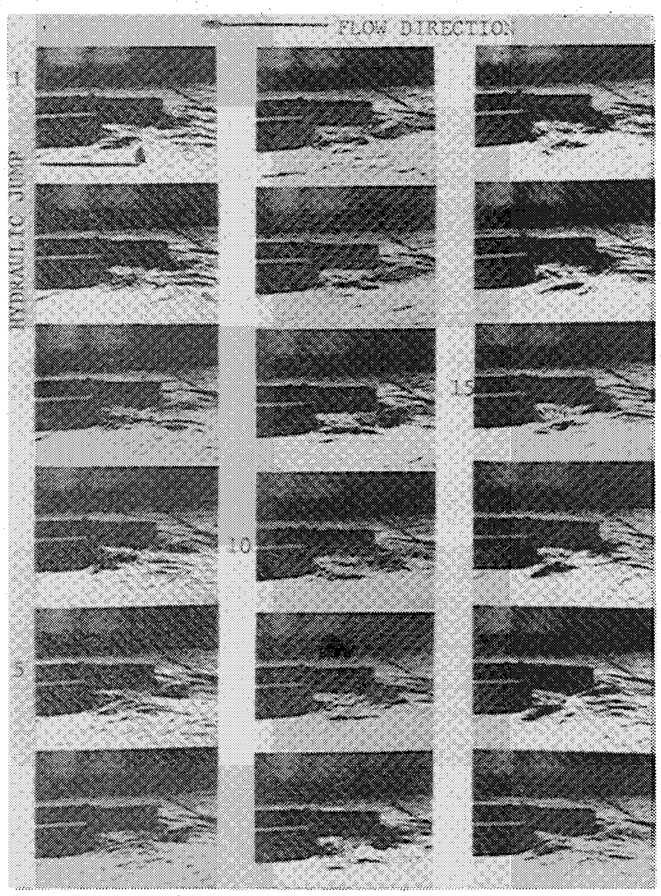

Fig. 8 Side view of wave oscillations on spike-tipped model.

L/D ratio of 2.25 at the end of the nozzle blocks with a compression angle of 16 deg (Fig. 3). This inclination allowed more mass entrainment into the cavity and induced strong "buzzing." The oscillations can be seen in Fig. 7. The figure shows about one cycle of wave oscillations recorded at 24 frames/s. Based on the water wave propagation velocity in shear layers, the frequency of oscillations f_w was calculated to be 0.95 cycle/s. Movie observation confirmed this value. In air for approximately the same conditions, the fundamental frequency of oscillations f_a calculated from Rossiter's equation[7] is 775 Hz which is three orders of magnitude larger. The calculation of the wave celerity in water and the speed of sound in air for the conditions used in the cavity tests (i.e., $h_\infty = 0.635$ cm, $Fr_\infty = M_\infty = 3$) also showed a three order-of-magnitude difference, $c_\infty = 0.25$ m/s and $a_\infty = 214$ m/s. Therefore, the ratio of oscillation frequency to wave speed in water is comparable to the ratio of oscillation frequency to sound speed in air, i.e., $f_w/c_\infty \sim f_a/a_\infty$. The hydraulic analogy is therefore valid. Figure 7 clearly shows the upstream motion of the dye before being expelled from the cavity, indicative of unsteady separated flow.

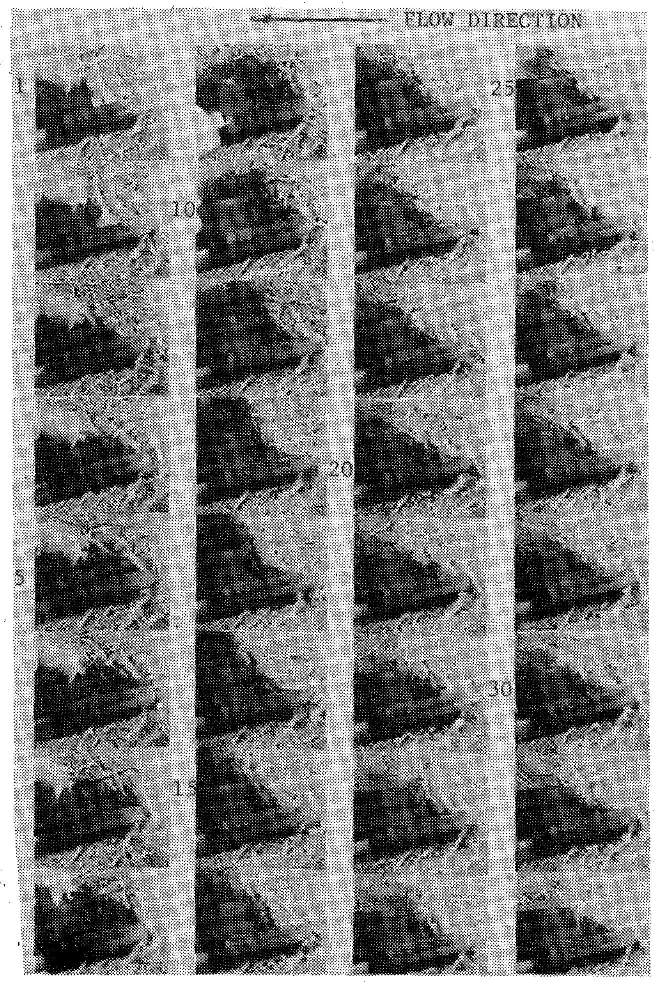

Fig. 9 Top view of wave oscillations on spike-tipped model.

Spike-Tipped Model

The hydraulic jump oscillations were generated by positioning the spike-tipped model 30.48 cm downstream of the nozzle blocks, in the middle of the freestream flow at an expansion angle of 12 deg. The expansion angle favors the formation of a separated area on the spike lee side and therefore generates a shear layer with an inflection point in the velocity profile, which is one of the conditions for buzzing to occur. Figures 8 and 9 show the "buzzing" phenomenon from the side and the top. In Fig. 8, one full cycle of oscillations is shown (some frames are omitted). It is clearly seen how the hydraulic jump changes position with respect to the spike. From experimental observation the frequency of oscillations f_w was calculated to be 0.8 cycle/s. For approximately the same conditions in air, the frequency of the first mode of oscillations f_a obtained in the experimental tests

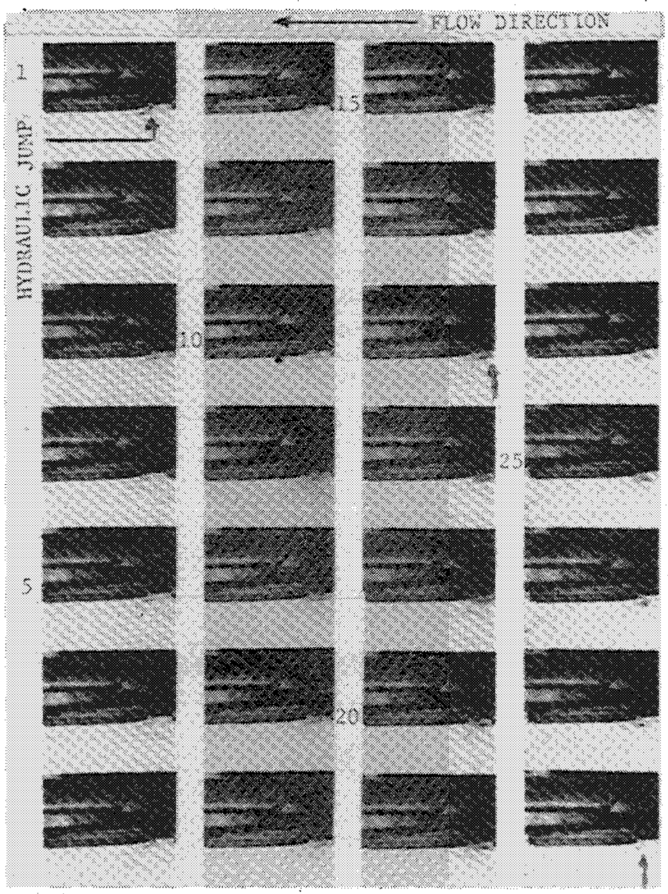

Fig. 10 Side view of wave oscillations on external compression inlet model.

was 1400 Hz.[8] Using the appropriate values for the wave propagation velocities, also in this case the same relation was obtained, i.e.,

$$f_w/c_\infty \sim f_a/a_\infty$$

In Fig. 9, two cycles of oscillations are shown. Dye is used to show the fluid oscillatory motion due to the wave motion (compare Fig. 9, frames 9 and 16 and frames 24 and 32).

Inlet

The "buzzing" for the inlet was generated by positioning the inlet model in the middle of the nozzle blocks at an expansion angle of 12 deg, which favors the formation of a separated shear layer on the external ramp of the inlet centerbody. Figure 10 clearly shows one cycle of the wave motion from the side of the inlet. The water waves buzz from the cowl lip almost to the tip of the centerbody. The calculated frequency of oscillations f_w was 0.25 cycle/s. The experiment of Ref. 9, performed in air, shows for high buzz a frequency of oscillations f_a of 360 Hz and for low buzz of about 150 Hz. For the latter condition, again $f_w/c_\infty \sim f_a/a_\infty$.

Figure 11 shows a top view of the buzzing phenomenon. The dye used moves slowly downstream in the inlet but the wave oscillations force it to be spilled from the cowl (compare frames 2 and 12). Also in frames 26-32 an upstream traveling wave is clearly visible.

Correlation

It is useful to correlate results obtained in water and air to the corresponding Strouhal numbers based on length, $S_\ell = fL/U_\infty$. For the experiments in air, the conditions selected for the correlation are those used in Refs. 8-10 for the spike-tipped, the inlet, and the cavity models, respectively. The Strouhal number calculated in water was $S_\ell = 0.3$ for the

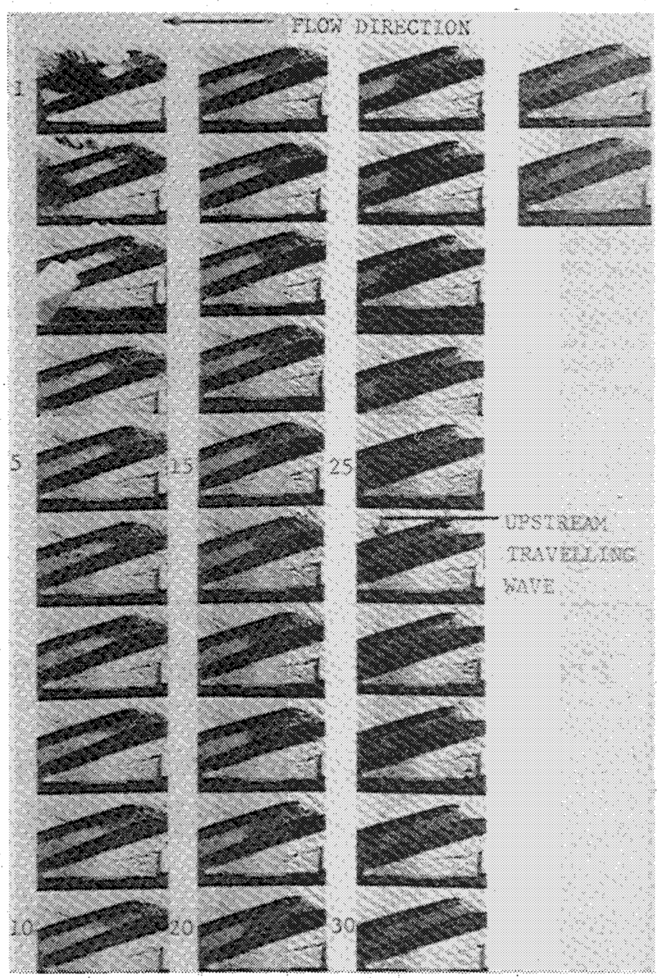

Fig. 11 Top view of wave oscillations on external compression inlet model.

cavity, 0.16 for the spike-tipped, and 0.17 for the inlet model. In air, the Strouhal number based on fundamental frequencies was 0.3 for the cavity, 0.16 for the spike-tipped, and 0.18 for the inlet model.

Conclusions

An experimental investigation of self-excited oscillations in water was performed on three different models, namely, a rectangular cavity, a spike-tipped blunt body, and an external compression inlet. The objective was to reproduce in water at a very low cost the "buzzing" phenomenon observed under special conditions in air and to take advantage of the hydraulic analogy to compare the results obtained in the two media. Data were obtained by using high-speed and real-time movies. A stability analysis was developed and a numerical solution obtained. The conclusions are as follows:

1) To obtain strong "buzzing" in water, the models had to be positioned at an inclination angle with the freestream flow.

2) The frequency of oscillations in water and air were consistent with the hydraulic analogy and the ratio of oscillation frequency to wave speed in water was approximately equal to the ratio of oscillation frequency to sound speed in air. The Strouhal numbers for air were comparable to those for water.

3) Photographic results show sustained self-excited oscillations and unsteady separated reversed flow by means of dye injections.

4) The numerical solution of the phenomenon shows that the disturbance, which is a Rayleigh instability, diminishes with increasing Froude number and nearly disappears at approximately $Fr = 3.5$. At supercritical propagation velocities, multiple small-amplification modes are observed.

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